

B.Sc 2nd Sem (Or) (By Asha Barman)

unit V (Maxwell's equations and EM wave)

Equation of continuity of current

Definition of current density ( $\vec{J}$ )

The amount of electric current flowing per unit cross-sectional area of a material is called current density ( $\vec{J}$ ). It is a vector quantity. Its direction being that of the motion of the charges at that point.

SI unit  $\rightarrow$  amp/m<sup>2</sup>

Before deriving this equation, students you should have some idea about surface and volume integrals. These are nothing but integration that you got earlier.

Surface integral - If we take integration about any surface, that integration is called surface integral. It is defined as —

$$\int_S \mathbf{v} \cdot d\mathbf{a}$$



where  $\int_s$  denotes → surface integration

$v \rightarrow$  any  $\vec{v}$  vector function

$da \rightarrow$  infinitesimal area.

Since the integration is called surface integral  
so here we will integrate around some  
area  $da$ .

volume integral -  $\int_V$  is denoted as -

$$\int_V T d\tau \quad \text{where } d\tau \rightarrow \text{infinitesimal volume}$$

$$\text{and } d\tau = dx dy dz$$

in cartesian co-ordinate

Since volume = length  $\times$  breadth  $\times$  height

$$\text{so } d\tau = dx dy dz$$

$T \rightarrow$  is any scalar function

$\int_V$  means integration around some volume.

Theorem to convert surface to volume or vice-versa

$$\int_V (\nabla \cdot v) d\tau = \int_S v \cdot da$$

where  $\nabla \cdot v$  is called divergence.

That means to convert  $\int_S v \cdot da$  any



surface integral to volume integral, we have to take the divergence of that vector.

Okay now let us derive the equation.

### Continuity Equation

We know current  $I = \frac{dq}{dt}$   $\left[ \because I = \frac{q}{t} \right]$

Or we can write

$$I = \frac{dq}{dt} = \int_S j \cdot d\mathbf{s} \quad \text{where } J \rightarrow \text{current density}, \quad \text{so } ds \rightarrow \text{surface area} \quad (1)$$

Now let us consider a closed surface  $S$  enclosing a volume  $V$ . If  $\rho \rightarrow$  volume density of charge

then total charge

$$q_V = \int_V \rho dV \quad \left[ \because \rho = \frac{q}{V} \right] \quad V \rightarrow \text{volume}$$

$\therefore (1) \Rightarrow$

[But here we use volume integral]

$$\frac{d}{dt} \int_V \rho dV = \int_S J \cdot d\mathbf{s}$$

$$\Rightarrow q_V = \rho V$$

Since electric charge can neither be created nor destroyed (i.e. the charge is conserved) it follows that the net flow of charge must be equal to the rate of decrease of the total charge inside the volume.

$$\text{i.e. } \int_S \mathbf{J} \cdot d\mathbf{s} = - \frac{d}{dt} \int_V \rho dV$$

(Here negative sign represents decrease of current)

$$\text{or } \int_S \mathbf{J} \cdot d\mathbf{s} = - \int_V \frac{\partial \rho}{\partial t} dV \quad \dots \quad (2)$$

now on transforming the surface integral into volume integral by Divergence theorem (as mentioned earlier)

$$\therefore \int_S \mathbf{J} \cdot d\mathbf{s} = \int_V (\nabla \cdot \mathbf{J}) dV$$

$$\text{Now (2)} \Rightarrow \int_V (\nabla \cdot \mathbf{J}) dV = - \int_V \frac{\partial \rho}{\partial t} dV$$



$$\Rightarrow \int_V \left( \nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} \right) dV = 0$$

This integral must be zero for any arbitrary volume  $V$ . Hence the integrand must vanish identically.

i.e

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

Or

$$\boxed{\text{div } \mathbf{J} + \frac{\partial \rho}{\partial t} = 0}$$

$\nabla \cdot \mathbf{J}$  divergence of  $\mathbf{J}$  we can write as

$\nabla$  div  $\mathbf{J}$  also.

This equation is known as equation of continuity.